

NONLINEAR EVEN AND ODD DISPLACED NUMBER STATE

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1. INTRODUCTION

Recent developments in quantum optics have led to suggestions of how non-classical states of the electromagnetic field may be prepared. In this sense, the displaced number states was proposed in literature by Oliveira et al. [1]. They have shown that this state can be generated through the number state $|n\rangle$ by the action of a displacement operator; for the microwave micromaser state this could be implemented by the action of a classical current that drives the cavity field.

Generalizations of the displaced number state (DNS) becomes now viable to be obtained. Recently it was proposed and investigated the statistical properties of the state of superposition of N displaced number states [2]. Interesting cases are obtained considering $N = 2$, which is the superposition of two DNS with the phase difference $\varphi = \pi$ between them, called even and odd displaced number states, which are the generalization of even and odd coherent states [3]. The nonlinear displaced number state of light field is another new state proposed recently [4], that is also a generalization of the DNS. Among several important properties that this state shows, we can cite the number-phase squeezing.

In this work we shall define the nonlinear even and odd displaced number states by introducing the nonlinearity. The idea of the nonlinearity of quantum states of the electromagnetic field can be found in several recent works in the literature [4-6]. Among these, we can cite the nonlinear even and odd coherent states [6], defined as the eigenstates of the operator $f(\hat{n})\hat{a}^2$

$$f(\hat{n})\hat{a}^2|f, \alpha\rangle = \alpha|f, \alpha\rangle \quad 01$$

whose coefficients are given by [6]

$$C_m = N_f \frac{\alpha^m}{\sqrt{m!} f(m)!} \quad 02$$

where

$$N_f = \left[\sum_{m=0}^{\infty} \frac{|\alpha|^{2m}}{m! f(m)!} \right]^{-1/2} \quad 03$$

with the convention

$$f(m)! = f(0)f(1)\dots f(m)$$

04

The investigation of the quantum statistical properties of the nonlinear even and odd displaced number states will show that this state presents many interesting quantum properties.

2. NONLINEAR EVEN AND ODD DISPLACED NUMBER STATES

The nonlinear even and odd displaced number state $|f, \pm\alpha, n\rangle$ is obtained by passing the even and odd displaced number states $|\pm\alpha, n\rangle$ through a nonlinear medium. The even and odd displaced number states is obtained as the superposition of two displaced number states

$$|\pm\alpha, n\rangle = N(|\alpha, n\rangle \pm |-\alpha, n\rangle) \quad 05$$

where $|\alpha, n\rangle$ is a Displaced Number State (DNS), defined as [1]

$$|\alpha, n\rangle = \hat{D}(\alpha)|n\rangle \quad 06$$

where $\hat{D}(\alpha)$ is the Glauber displacement operator, given by [7].

In Eq.(5) we can see that the initial number of photon of the DNS defines the parity of the state. The + sign in Eq.(5) corresponds to even displaced number state (EDNS) and the - sign corresponds to odd displaced number state (ODNS). The coefficients the superposition are given by

$$C_m^\pm = N \begin{cases} \sqrt{\frac{n!}{m!}} e^{-\alpha^2/2} \alpha^{(m-n)} L_n^{(m-n)}(\alpha^2) [1 \pm (-1)^{m-n}], & m \geq n \\ \sqrt{\frac{m!}{n!}} e^{-\alpha^2/2} \alpha^{(n-m)} L_m^{(n-m)}(\alpha^2) [1 \pm (-1)^{n-m}], & n \geq m \end{cases} \quad 07$$

with α real, N the normalization constant and $L_n^{(k)}(x)$ is the Laguerre polynomial of order n.

We can obtain the coefficients of the nonlinear even displaced number state (NLEDNS) and nonlinear odd displaced number state (NLODNS), $C_m^{\pm NL}$, in terms of the coefficients of the EDNS and ODNS given by Eq.(7) as

$$C_m^{\pm NL} = \frac{NC_m^\pm}{f(m)!} \quad 08$$

with $f(0)=1$.

3. PHOTON NUMBER DISTRIBUTION OF THE NLEDNS AND NLODNS

The photon number distribution of the NLEDNS and NLODNS $P_m^{\pm NL}$ are given in terms of its coefficients $C_m^{\pm NL}$ of the number distribution given in Eq.(08). In Fig.(1) displays the photon number distribution $P_m^{\pm NL}$ of the NLODNS, with the initial number of photons $n=1$, displacement parameter $\alpha=3$ and the nonlinear function $f(m)$ given by

$$f(m) = (1 + km)^{-1} \quad 09$$

where k is the intensity of the nonlinearity to set a) $k=0$ and b) $k=0,05$. As Fig.(1) shows, increasing the intensity of the nonlinearity, the oscillations in the photon

number distribution, present in the ODNS, becomes an distribution with the photon number better defined.

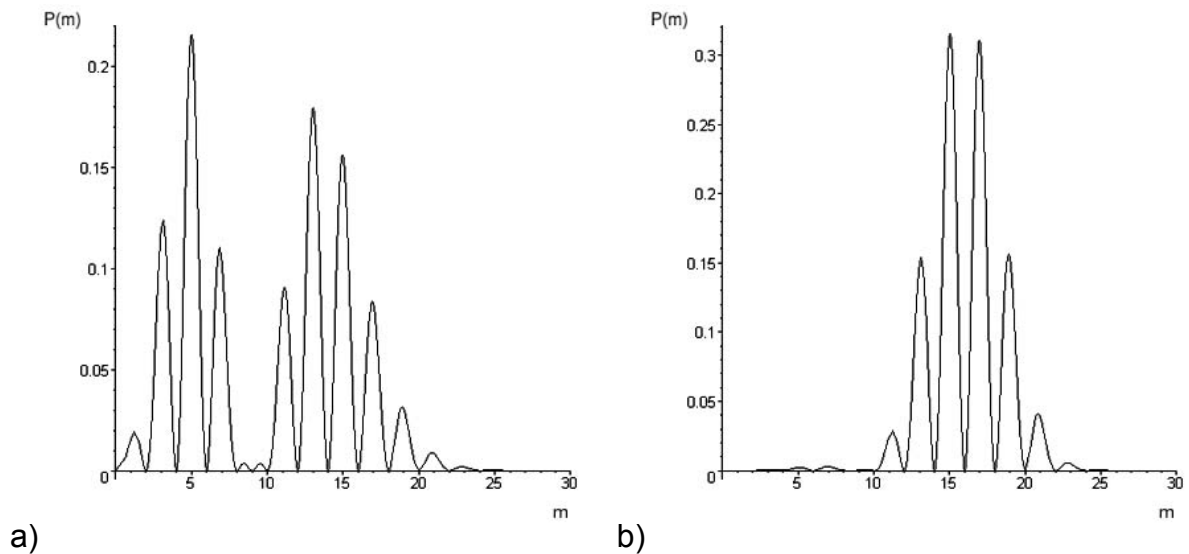


Figure 01 – Photon number distribution for the NLDNS a) $k = 0$ and b) $k = 0,05$.

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