Teleporting entanglements of cavity-field states

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(Received 15 May 2004; published 23 August 2004)

We present a scheme to teleport an entanglement of zero- and one-photon states from one cavity to another. The scheme, which has 100% success probability, relies on two perfect and identical bimodal cavities, a collection of two kinds of two-level atoms, a three-level atom in a ladder configuration driven by a classical field, Ramsey zones, and selective atomic-state detectors.

DOI: 10.1103/PhysRevA.70.025803 PACS number(s): 42.50.Dv, 03.65.Ud

I. INTRODUCTION

Since the proposition by Bennett *et al.* [1], the teleportation phenomenon has received great attention, and a number of protocols have been suggested to implement the process in different contexts, such as for trapped ions [2], running waves [3–5], and cavity QED [6]. Experimentally, teleportation has been demonstrated for discrete variables [7–9] and for a single-mode electromagnetic field of continuous variables [10]. The main experimental challenge consists in the so-called Bell-state measurement, performed on the Bell operator basis for the particle whose state is to be teleported plus its partner composing the quantum channel [1]. In the experiment of Bowmeester *et al.* [7], designed to manipulate only the polarization state of single-photon pulses, only one of the four Bell states is discriminated, resulting in a success rate of 25%. In the experiment of Boschi *et al.* [8], employing the entanglement between the spatial and polarization degrees of freedom of a photon, it was possible to distinguish the four Bell states, allowing a success rate of 100%. In the experiment of Kim *et al.* [9], following exactly the original protocol by Bennett *et al.* [1] and taking advantage of nonlinear interactions to accomplish the Bell-state measurements, a success rate of 100% can also be achieved in principle for teleporting a polarization state. Teleportation of continuous variables was demonstrated in the experiment of Furusawa *et al.* [10], with a fidelity of 0.58 ± 0.02, higher than the critical 0.5 classical bound attainable in the absence of quantum correlation. Teleportation of an entangled qubit with a success rate of 25% was recently reported [11], and efforts toward 50% success probability are in progress [11].

Teleportation of entanglements employing linear optical elements, at the expense of decreasing the success probability to 50%, has also been studied [5,12], and a proposal for achieving high fidelity in the process of teleportation of entangled states in running waves has recently been addressed [13], with success probability near 100%. Teleportation (and decoherence) of entangled coherent states in running waves has also been studied [14], and a success probability of 50% was found.

In the realm of cavity QED, schemes have been proposed for teleportation of two-particle entangled states [15], multiparticle entangled atomic states [6], and also entangled field states inside high-Q cavities [16]. In Ref. [16] the authors propose a scheme to teleport an entangled state composed of two qubits (readily generalized for teleportation of an N-qubit field state) from a pair of high-Q cavities to another pair of high-Q cavities. As usual, Ramsey zones and the Stark shift effect are employed. Also, the authors suggest to employ long-lived Rydberg atoms in circular states with principal quantum number around 50 for the controlled atom-field interaction times, in such a way that the whole losses due to atomic spontaneous emission and dissipation in high-Q cavities were neglected.

Here we propose a scheme to teleport a two-mode entanglement of zero- and one-photon states from one cavity to another. Unlike the scheme in [16], in which all six high-Q cavities required to perform the teleportation contain a single mode, our scheme just needs two high-Q (identical) bimodal cavities [17]. Also, our scheme requires a collection of two kinds of two-level atoms, a three-level atom in a ladder configuration driven by a classical field, Ramsey zones, and selective atomic-state detectors. The success probability is 100%. As assumed in [16], all the losses due to atomic spontaneous emission and dissipation in the cavities are neglected. In fact, since the decoherence time is of the same order as the lifetimes of the qubits in a high-Q cavity and as the (spontaneous) atomic decay, the experimental implementation of the present scheme should be realized during 10^{-2} s, a typical time for both decoherence and damping of atomic and cavity qubits [18].

II. CONTROLLED INTERACTIONS

To perform teleportation of zero- and one-photon entangled states between two high-Q cavities we will need the following operators:

$$H_{on} = \hbar\lambda(\sigma^- a^\dagger + \sigma^+ a), \tag{1}$$

$$H_{off} = \frac{\hbar\lambda^2}{\delta} a^\dagger a |e\rangle\langle e|, \tag{2}$$

$$R_\pm = \sqrt{1/2}(I \pm i\sigma_y), \tag{3}$$

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$$H_c = \hbar\chi(a^\dagger b + ab^\dagger). \tag{4}$$

Equation (1) is the usual Jaynes-Cumming model [19] and describes a resonant interaction of mode a represented by creation and annihilation operators $(a, a^\dagger)$ with a two-level atom represented by the Pauli operators σ; λ is the atom-field coupling parameter. Equation (2) stands for the dispersive atom-field interaction [19] and can be implemented via a Stark shift; $\delta = (\omega - \omega_0)$ is the detuning between the field frequency ω and the atomic frequency ω_0. Equation (3) represents the action of the Ramsey zone properly adjusted to produce either R_+ or R_- [18]. Equation (4) was recently obtained for both a three-level atom in ladder configuration and a two-level atom interacting with two modes of a cavity field [20], χ being the coupling between the three- or two-level atom and the field modes. Using the operators defined in Eqs. (1)–(4), it is easy to verify the following evolutions:

$$R_\pm \begin{cases} |g\rangle \to \sqrt{1/2}(|e\rangle \pm |g\rangle) \equiv |\pm\rangle, \\ |e\rangle \to \sqrt{1/2}(|e\rangle \mp |g\rangle) \equiv |\mp\rangle, \end{cases} \tag{5}$$

$$U_{on}(\alpha,\beta) \begin{cases} |g\rangle_\alpha|0\rangle_\beta \to |g\rangle_\alpha|0\rangle_\beta, \\ |e\rangle_\alpha|0\rangle_\beta \to -i|g\rangle_\alpha|1\rangle_\beta, \\ |g\rangle_\alpha|1\rangle_\beta \to -i|e\rangle_\alpha|0\rangle_\beta, \end{cases} \tag{6}$$

$$U_{off}(\alpha,\beta) \begin{cases} |+\rangle_\alpha|1\rangle_\beta \to -|-\rangle_\alpha|1\rangle_\beta, \\ |-\rangle_\alpha|1\rangle_\beta \to -|+\rangle_\alpha|1\rangle_\beta, \\ |+\rangle_\alpha|0\rangle_\beta \to |+\rangle_\alpha|0\rangle_\beta, \\ |-\rangle_\alpha|0\rangle_\beta \to |-\rangle_\alpha|0\rangle_\beta, \end{cases} \tag{7}$$

$$U_c(a,b) \begin{cases} |0\rangle_a|1\rangle_b \to C_1|0\rangle_a|1\rangle_b + iC_2|1\rangle_a|0\rangle_b, \\ |1\rangle_a|0\rangle_b \to C_1|1\rangle_a|0\rangle_b + iC_2|0\rangle_a|1\rangle_b. \end{cases} \tag{8}$$

We note that the evolutions $U_{on}(\alpha,\beta)$ and $U_{off}(\alpha,\beta)$ are obtained by adjusting the interaction times as $\lambda t = \pi/2$ and $\lambda^2 t/\delta = \pi$ from $U_{on}(\alpha,\beta) = \exp[-iH_{on}t/\hbar]$ and $U_{off}(\alpha,\beta) = \exp[-iH_{off}t/\hbar]$, respectively. The subindex α stands for atoms in the ground and excited states (g, e), and β represents the modes (a, b) and cavities (1,2).

Here we are assuming that the operation $U(1,1a)$ leaves mode $1b$ unaffected. Indeed, this is a crucial point, since relative phases will necessarily appear from atom-field interactions. Next, let us prove that these phases are irrelevant. As is readily seen from the Hamiltonian model for a two-level atom interacting resonantly with, say, mode b and off-resonantly with mode a, the effective Hamiltonian leads to an energy shift corresponding to mode a. This shifting is inversely proportional to the detuning $\delta = (\omega_a - \omega_0)$ between the field frequency ω_a of mode a and the atom frequency ω_0 [see Eq. (2)]. Thus, if δ is large enough compared with the atom-field coupling parameter λ, we can neglect the effect of the off-resonant interaction. To estimate the additional phases gained due to the dispersive interaction between one of the two modes (ω_a, ω_b) of the cavity and the atomic frequency of the two-level atom, let us consider the atom-field interactions such that $(\omega_0 = \omega_b > \omega_a)$ and $\lambda t = \pi/2$. Thus, the mode a will be shifted by the phase $\phi = \lambda^2 t/(\omega_0 - \omega_a) = \pi\lambda/2(\omega_0 - \omega_a)$. Next, let us suppose a Stark shift on mode b leading to $\omega_b = \omega_0 + \delta$ such that $\lambda^2 t/\delta = \pi$. In this case, mode a will be shifted by the phase $\phi' = \lambda^2 t/[(\omega_0 - \omega_a) \pm \delta]$. Taking the experimental values [18] $\omega_0 - \omega_a \simeq 10^{10}$ Hz, $\lambda \simeq 10^5$ Hz, and $\delta = 10\lambda$, then $\phi \simeq \phi' \simeq 10^{-5}$ rad, which can be safely neglected.

III. TELEPORTATION

A. Preparing the state to be teleported in cavity 1

First, a two-level atom 1, $|e\rangle_1$, interacts resonantly with mode a, leaving cavity 1 with one photon. Then, a three-level atom [21] completes the preparation by entangling modes a and b (in this stage there is no further use for the atoms, so they are thrown away):

$$U_{on}(1,1a)|e\rangle_1|0\rangle_{1a}|0\rangle_{1b} \to |g\rangle_1|1\rangle_{1a}|0\rangle_{1b}, \tag{9}$$

$$U_c(1a,1b)|1\rangle_{1a}|0\rangle_{1b} \to (C_1|1\rangle_a|0\rangle_b + C_2|0\rangle_a|1\rangle_b) \equiv |\psi\rangle_{1a1b}, \tag{10}$$

$$|\psi\rangle_{1a1b} = (C_1|\overline{0}\rangle_{1a1b} + C_2|\overline{1}\rangle_{1a1b}), \tag{11}$$

where we have defined $|\overline{0}\rangle_{ij} \equiv |1\rangle_i|0\rangle_j, |\overline{1}\rangle_{ij} \equiv |0\rangle_i|1\rangle_j$, and incorporated the factor i in the constant C_2.

B. Preparing the nonlocal channel

The nonlocal channel is composed of cavity 2 and a two-level atom 2. The same procedure described to prepare the state to be teleported in cavity 1 is now repeated for cavity 2 with the modification $|C_1| = |C_2| = 1/\sqrt{2}$. Next, the two-level atom 2, $|e\rangle_2$, crosses a Ramsey zone R_- and cavity 2 interacts off-resonantly [$U_{off}(2,2b)$ with mode b], as indicated by Eq. (5) and Eq. (7). The result is

$$|\chi\rangle_{NL} = \frac{1}{\sqrt{2}}(|+\rangle_2|1\rangle_{2a}|0\rangle_{2b} - i|-\rangle_2|0\rangle_{2a}|1\rangle_{2b}) \equiv \frac{1}{\sqrt{2}}(|+\rangle_2|\overline{0}\rangle_{2a2b} - i|-\rangle_2|\overline{1}\rangle_{2a2b}). \tag{12}$$

The undesirable phase $(-i)$ can easily be eliminated from the nonlocal channel by sending an excited two-level atom interacting off-resonantly with mode a (or mode b) with the interaction time adjusted to $\lambda^2 t/\delta = \pi/2$. After that, the state describing the whole system composed of atom 2, cavity 1, and cavity 2 reads

$$|\Psi\rangle_T \equiv |\chi\rangle_{NL}|\psi\rangle_{1a1b} = \frac{1}{\sqrt{2}}\{C_1|\overline{0}\rangle_{1a1b}|+\rangle_2|\overline{0}\rangle_{2a2b} - C_1|\overline{0}\rangle_{1a1b}|-\rangle_2|\overline{1}\rangle_{2a2b} + C_2|\overline{1}\rangle_{1a1b}|+\rangle_2|\overline{0}\rangle_{2a2b} - C_2|\overline{1}\rangle_{1a1b}|-\rangle_2|\overline{1}\rangle_{2a2b}\}. \tag{13}$$

If we define the states of the Bell basis composed of atom 2 and modes a, b of cavity 1 as

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|\overline{0}\rangle_{1a1b}|+\rangle_2 \pm |\overline{1}\rangle_{1a1b}|-\rangle_2), \tag{14}$$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\overline{0}\rangle_{1a1b}|-\rangle_2 + |\overline{1}\rangle_{1a1b}|+\rangle_2), \tag{15}$$

Eq. (13) can be expanded as

$$|\Psi\rangle_T = \frac{1}{\sqrt{2}}\{(C_1|\overline{0}\rangle_{2a2b} - C_2|\overline{1}\rangle_{2a2b})|\Psi^+\rangle + (C_1|\overline{0}\rangle_{2a2b} + C_2|\overline{1}\rangle_{2a2b})|\Psi^-\rangle + (C_2|\overline{0}\rangle_{2a2b} - C_1|\overline{1}\rangle_{2a2b})|\Phi^+\rangle - (C_2|\overline{0}\rangle_{2a2b} + C_1|\overline{1}\rangle_{2a2b})|\Phi^-\rangle\}. \tag{16}$$

C. Bell measurement

To complete the teleportation process, we have to perform a joint measurement on the state $|\Psi\rangle_T$ involving both atom 2 and cavity 1 as represented by one of the four results $|\Psi^+\rangle$, $|\Psi^-\rangle$, $|\Phi^+\rangle$, or $|\Phi^-\rangle$. To do that, the atom 2 is left to interact off-resonantly with mode b in cavity 1 and then crosses R_+ as indicated by Eq. (2) and Eq. (3). When the state $|\Psi^{\pm}\rangle$ is found the result is

$$|\Psi^{\pm}\rangle \rightarrow \frac{1}{\sqrt{2}}(|\overline{0}\rangle_{1a1b} \pm |\overline{1}\rangle_{1a1b})|e\rangle_2. \tag{17}$$

On the other hand, when finding the state $|\Phi^{\pm}\rangle$ we have

$$|\Phi^{\pm}\rangle \rightarrow \frac{1}{\sqrt{2}}(|\overline{0}\rangle_{1a1b} \pm |\overline{1}\rangle_{1a1b})|g\rangle_2. \tag{18}$$

Thus, measuring atom 2 allows one to distinguish between the states $|\Psi^{\pm}\rangle$ and $|\Phi^{\pm}\rangle$. Next, we have to discern the phase ($\pm$). This is done by sending successively two more two-level atoms in the ground state ($|g\rangle_a, |g\rangle_b$) resonantly with modes a, b. The atom $|g\rangle_a$, after interacting resonantly with mode a as indicated by Eq. (1), crosses R_+, and evolves according to Eq. (3). After that, atom $|g\rangle_b$ enters the cavity, interacting resonantly with mode b as indicated by Eq. (1) and crosses R_-, evolving also in accord with Eq. (3). The result is

$$(|\overline{0}\rangle_{1a1b} \pm |\overline{1}\rangle_{1a1b}) \rightarrow \begin{cases} \frac{1}{\sqrt{2}}(|e\rangle_a|e\rangle_b + |g\rangle_a|g\rangle_b) & \text{if} \quad (+), \\ \frac{1}{\sqrt{2}}(|e\rangle_a|g\rangle_b + |g\rangle_a|e\rangle_b) & \text{if} \quad (-). \end{cases} \tag{19}$$

Therefore, when the atoms $|g\rangle_a$ and $|g\rangle_b$ are measured both in either the ground or the excited state, the phase is (+); if one of them is found to be in the ground state while the other is in the excited state, the phase is (−). We note from Eq. (16) that the only prompt result that completes the teleportation process from cavity 1 to cavity 2 is a joint measurement resulting in the state $|\Psi^-\rangle$. If the joint measurement results in $|\Psi^+\rangle$, an atom interacting dispersively with mode a (or mode b), as indicated by Eq. (2), can be adjusted ($\lambda^2 t/\delta = \pi$) to repair the phase: $(C_1|\overline{0}\rangle_{2a2b} - C_2|\overline{1}\rangle_{2a2b})|e\rangle_b \rightarrow (C_1|\overline{0}\rangle_{2a2b} + C_2|\overline{1}\rangle_{2a2b})|e\rangle_b$, the atom $|e\rangle_b$ dropping out. When a joint measurement leads to the state $|\Phi^-\rangle$, the teleportation process can be completed by sending a three-level atom as indicated by Eq. (8), with C_1 equal to zero. On the other hand, when the joint measurement results in $|\Phi^+\rangle$ the previous procedure must be followed by a two-level atom interacting dispersively [cf. Eq. (2) with $\lambda^2 t/\delta = \pi$], which completes the teleportation process.

IV. CONCLUSIONS

We have presented a scheme to teleport an entanglement of zero- and one-photon states from one high-Q cavity to another. Our scheme is based on the following operations: (i) control of the interaction times between two-level atoms and one of the two modes sustained by the two high-Q cavities; (ii) entanglement of the two modes of the high-Q cavities (cf. Refs. [20,21]); (iii) use of Stark shifts and Ramsey zones. In addition, we have computed the relative phase gained by mode a when the interaction between the atom and mode b takes place. This phase is shown to be irrelevant. Assuming the time spent in experiments to be less than the typical times of damping for the qubits employed in this scheme, which are of the same order as the decoherence time of these qubits, allows one to neglect both atomic spontaneous decay and losses in the high-Q cavities. Our scheme is able to discern each one of the four Bell states with 100% success probability. The present scheme is more economical than others in the literature since it reduces the number of cavities necessary to perform the experiment; this diminishes the decoherence effect caused by unavoidable interactions between the system and its environment. In addition, it is more advantageous than those based on single-mode teleportation when it is desired that the final teleported state be entangled. In this case, just teleporting independently (and coherently) each cavity mode will require further operations to entangle the two modes, thus implying additional procedures and eventual sources of errors.

ACKNOWLEDGMENTS

We thank the VPG/Vice-Reitoria de Pós-Graduação e Pesquisa-UCG (N.G.A.), the CAPES (G.P.), and the CNPq (N.G.A., A.T.A., B.B), Brazilian Agencies, for partially supporting this work.

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